FALL 2019: MATH 558 QUIZ 12 SOLUTIONS

Each question is worth 5 points.

1. Let G be a group and H a subset of G. Define what it means for H to be a subgroup of G. Then give an example of a group G with subgroup H so that H is a proper subgroup of G i.e., $H \neq e$ and $H \neq G$.

Solution. $\mathrm{Sl}_2(\mathbb{R})$ is a proper subgroup of $\mathrm{Gl}_2(\mathbb{R})$. $2\mathbb{Z}$ is a proper subgroup of $\mathbb{Z}$.

2. Recall $S_3 = \{I, \tau, \tau^2, \sigma, \tau\sigma, \tau^2\sigma\}$, where $\tau^3 = \sigma^2 = I$ and $\sigma\tau = \tau^2\sigma$. Let $H = \{I, \tau, \tau^2\}$ and $K = \{I, \sigma\}$ be subgroups. Find:

(i) The distinct right cosets of H and the distinct right cosets of K, writing the elements in these sets in terms of the expressions given above.

(ii) Find a group element $g \in S_3$ such that $gK \neq Kg$. Justify your answer.

Solution. For part (i): $H = \{I, \tau, \tau^2\}$ and $H\sigma = \{\sigma, \tau\sigma, \tau^2\sigma\}$ are the distinct right cosets of H. The distinct right cosets of K are: $K = \{I, \sigma\}, K\tau = \{\tau, \sigma\tau\} = \{\tau, \tau^2\sigma\}$, and $K\tau^2 = \{\tau^2, \sigma\tau^2\} = \{\tau^2, \tau\sigma\}$.

For part (ii): Take $g = \tau$. Then $\tau K = \{\tau, \tau\sigma\}$ and $K\tau = \{\tau, \tau^2\sigma\}$, so the left coset of K with respect to τ is not equal to the right coset of K with respect to τ.